

Neutrino oscillations and the seesaw origin of neutrino mass

O. G. Miranda

*Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN
Apdo. Postal 14-740 07000 Mexico, Distrito Federal, Mexico*

J. W. F. Valle

*AHEP Group, Institut de Física Corpuscular – C.S.I.C./Universitat de València, Parc Científic de Paterna.
C/Catedrático José Beltrán, 2 E-46980 Paterna (València) - SPAIN*

Abstract

The historical discovery of neutrino oscillations using solar and atmospheric neutrinos, and subsequent accelerator and reactor studies, has brought neutrino physics to the precision era. We note that CP effects in oscillation phenomena could be difficult to extract in the presence of unitarity violation. As a result upcoming dedicated leptonic CP violation studies should take into account the non-unitarity of the lepton mixing matrix. Restricting non-unitarity will shed light on the seesaw scale, and thereby guide us towards the new physics responsible for neutrino mass generation.

Keywords: Neutrino mass, Neutrino mixing, Neutrino interactions

1. Introduction

Particle physics has seen two historic discoveries in less than twenty years: the confirmation of the mechanism of electroweak symmetry breaking [1] and the discovery of neutrino oscillations [2, 3, 4, 5, 6], both deservedly honored with the Nobel prize. The unification paradigm [7, 8] and the good behaviour of the electroweak breaking sector, including naturalness, perturbativity and stability [9], have so far provided a strong theoretical motivation for new physics. Other hints are the understanding of flavour and the unification with gravity, in addition to the challenges posed by cosmological observations associated to dark matter, dark energy and inflation. Last, but not least, the need to account for non-zero neutrino mass [10, 11] plays a key role in the quest for new physics [12].

The most popular mechanism of neutrino mass generation ascribes the smallness of neutrino mass as resulting from the exchange of heavy messenger particles, such as right-handed iso-singlet neutrinos and/or iso-triplet scalar bosons, known as the seesaw mechanism [12]. When formulated at low-scale this naturally implies new effects in neutrino propagation that go beyond the oscillatory behaviour, as explained below. In particular, future neutrino experiments will face the challenge of disentangling “conventional” CP violation with that associated to the non-unitarity of the lepton mixing matrix, which in turn results as an indirect effect of the extra neutral heavy right-handed neutrinos.

In what follows, we briefly review current neutrino oscillation parameters and describe novel effects associated to right-handed neutrino admixture in the charged current weak interaction, expected in low-scale seesaw schemes, purely in the context of the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ paradigm. We also recompile current limits on right-handed neutrino mass and mixing parameters.

2. Three neutrino mixing and oscillations

Generic neutrino mass schemes require interactions associated to new Yukawa couplings that do not commute with those of the charged leptons, leading to the phenomenon of mixing in the charged current weak interactions, analogous to the CKM mixing of quarks [13]. However, as we will see its structure can be richer.

2.1. Lepton mixing matrix for Dirac neutrinos

The mixing of leptons arising from the non-simultaneous diagonalizability of the Dirac neutrino and charged lepton mass matrices is given by an arbitrary unitary matrix

$$U = \omega_0(\gamma) \prod_{i < j}^3 \omega_{ij}(\eta_{ij}), \quad (1)$$

where each of the ω factors is effectively 2×2 , characterized by an angle and a corresponding CP phase, e.g.

$$\omega_{13} = \begin{pmatrix} c_{13} & 0 & e^{-i\phi_{13}} s_{13} \\ 0 & 1 & 0 \\ -e^{i\phi_{13}} s_{13} & 0 & c_{13} \end{pmatrix}, \quad (2)$$

while $\omega_0(\gamma)$ is an arbitrary diagonal unitary matrix. In complete analogy with the standard model quark sector we can use the phase redefinition freedom for neutral and charged leptons to show that only one independent CP phase remains for the three Dirac neutrinos. To find this rephasing invariant parameter we use the conjugation property [14]

$$P^{-1} U P = \omega_{23}(\theta_{23}; \phi_{23} - \beta) \omega_{13}(\theta_{13}; \phi_{13} - \alpha) \omega_{12}(\theta_{12}; \phi_{12} + \beta - \alpha), \quad (3)$$

with $P = \text{diag}(e^{i\alpha}, e^{i\beta}, 1)$, which allows us to identify [15],

$$\delta \leftrightarrow \phi_{13} - \phi_{12} - \phi_{23}. \quad (4)$$

The form of the lepton mixing matrix in this case is the same as the CKM matrix describing the mixing of quarks.

2.2. Lepton mixing matrix for Majorana neutrinos

The case of 3×3 unitary lepton mixing matrix arises, for example, if the light neutrino masses result from the exchange of iso-triplet scalar messengers through Type II seesaw [14] (see below). Within the symmetric parametrization the mixing matrix has the form

$$U = \omega_{23}(\theta_{23}; \phi_{23}) \omega_{13}(\theta_{13}; \phi_{13}) \omega_{12}(\theta_{12}; \phi_{12}), \quad (5)$$

where the diagonal unitary matrix $\omega_0(\gamma)$ is eliminated by rephasing the charged leptons but no rephasing of the neutrinos is possible, leading to two extra phases in the mixing matrix U compared with the previous case [14]. Explicitly, the matrix U can be written as:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}e^{-i\phi_{12}} & s_{13}e^{-i\phi_{13}} \\ -s_{12}c_{23}e^{i\phi_{12}} - c_{12}s_{13}s_{23}e^{-i(\phi_{23}-\phi_{13})} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{-i(\phi_{12}+\phi_{23}-\phi_{13})} & c_{13}s_{23}e^{-i\phi_{23}} \\ s_{12}s_{23}e^{i(\phi_{12}+\phi_{23})} - c_{12}s_{13}c_{23}e^{i\phi_{13}} & -c_{12}s_{23}e^{i\phi_{23}} - s_{12}s_{13}c_{23}e^{-i(\phi_{12}-\phi_{13})} & c_{13}c_{23} \end{pmatrix}. \quad (6)$$

Although they do not show up in oscillations, the ‘‘Majorana’’ phases will affect lepton number violation processes such as $0\nu\beta\beta$ [16, 17]. It has been noticed that this fully symmetrical presentation is more convenient for the description of $0\nu\beta\beta$ decay than the PDG form [15] (see below).

2.3. Status of the three neutrino picture

Neutrinos from natural sources like the Sun, and from the interaction of cosmic rays with the Earth's atmosphere gave us the first indications for neutrino conversion. Neutrinos are also produced in the laboratory, both at accelerators as well as nuclear reactors. The disappearance of muon neutrinos over a long-baseline probing the same region of squared mass splitting relevant for atmospheric neutrinos has been obtained in accelerator neutrino oscillation experiments, starting with the KEK to Kamioka (K2K) neutrino oscillation experiment, the MINOS Experiment using the NuMI Beam-line facility at Fermilab, and currently the T2K (Tokai to Kamioka) experiment in Japan and the NOvA experiment in the USA. These have also substantially helped in determining the neutrino parameters with a high level of precision [18, 19, 20, 21, 22].

The results of solar neutrino experiments have also been confirmed by reactor neutrino studies at the KamLAND experiment [6]. More sensitive experiments such as Double Chooz [23], RENO [24], and Daya Bay [25] have confirmed that θ_{13} is nonzero. Particularly Daya Bay has now provided a precision measurement of θ_{13} [25], one of the most important results in the field in this decade. Altogether, neutrino physics is now a mature branch of

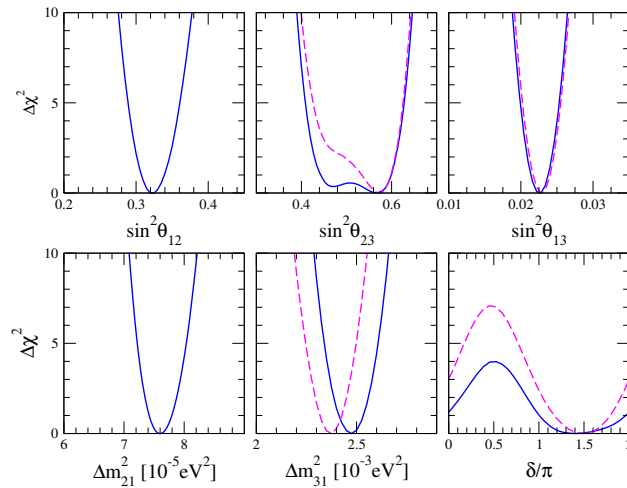


Figure 1: Neutrino mixing angles and squared mass differences, as determined from the global analysis in Ref. [11]. The normal hierarchy case is shown with solid lines; dashed lines are used for inverted hierarchy.

science, in which the three neutrino mixing angles as well as the two squared mass differences have been determined with high precision. The current status of the determination of the solar, atmospheric, reactor and accelerator neutrino oscillation parameters is summarized in Fig. (1), taken from Ref. [11], where the solid and dashed curves refer to the normal and inverted mass ordering. These include data from a number of solar neutrino experiments [26, 27, 28, 29, 30, 31, 32, 33, 34, 35] and atmospheric data from the Super-Kamiokande experiment, described in Ref. [36]. All of these experiments have been taken into account in the results summarized here, as well as in other similar studies, all of which agree at the 3σ level [37, 38]. One sees that for the case of θ_{23} there is still room for improvement. Concerning the standard Dirac CP phase, δ in Eq. (4), at the moment there is only a hint that it is non-vanishing. Finally, notice that oscillation experiments provide no information on the absolute neutrino mass scale, nor on the values of the Majorana phases.

2.4. Robustness of neutrino oscillations

How robust is the oscillation interpretation of current neutrino data? So profound is the discovery of neutrino oscillations and the determination of neutrino oscillation parameters that it requires careful consideration of any possible loopholes. The good agreement between the standard solar model sound speed profile and helioseismology results

substantially constrain possible astrophysical uncertainties [39, 40]. Yet the effect of varying solar neutrino fluxes has been widely discussed, without substantial impact on the neutrino oscillation parameters. However, although experiments are now measuring neutrino fluxes to within a few percent, helioseismic studies have reached accuracies of about a few parts in a thousand. Hence, it is not inconceivable that discrepancies might eventually show up [39, 40].

Uncertainties associated with the possibility of solar density fluctuations were first suggested in the late nineties [41, 42]. Such fluctuations deep within the Sun could have a resonant origin from magnetic fields in the radiative zone [43]. Direct helioseismic tests are not necessarily in conflict with such variations, since they are not sensitive to fluctuations with size around several hundreds of kilometers to which neutrino oscillations are sensitive [44, 45]. Indeed, it has been shown that the effect on solar neutrino oscillations can be important. However, the measurement of neutrino properties at KamLAND provides valuable independent information which can, in fact, be used to probe the deep solar interior [46]. Solar neutrino measurements from SNO are now sufficiently precise that neutrino oscillation parameters can be inferred independently of any assumptions about fluctuation size [47]. In fact the fluctuation amplitudes above 5% now excluded if their correlation lengths lie in the range of several hundred km.

Magnetic fields in the solar convective zone can cause spin-flavour precession and produce a solar $\bar{\nu}_e$ flux [48, 49]. The robustness of the oscillation hypothesis has also been analysed in this context. It has been shown that $\bar{\nu}_e$ production can be greatly enhanced for the case of random magnetic fields [50]. The search for anti-neutrinos from the Sun can be used to constrain the neutrino magnetic moments [51]. In summary, laboratory oscillation studies not only give a crucial confirmation of the solar neutrino oscillation hypothesis, ruling out exotic solutions, but also establish the robustness of large mixing angle oscillations [52, 53].

3. Seesaw paradigm

As the only electrically neutral standard model fermions, neutrino mass generation could be different from the standard Higgs mechanism. In particular neutrinos could be light as a result of their Majorana nature. Indeed, unless prevented by basic symmetries such as electric or colour charge [14], fermions are intrinsically two-component objects. The emergence of Dirac neutrinos, would then signal extra symmetry assumptions. For example one could assume lepton number conservation directly, or some extended symmetry that implies the conservation of lepton number, as some specific flavour symmetries [54]. Moreover, following Weinberg, we note the simplest operator capable of inducing neutrino masses is a unique dimension-5 operator, generally implying lepton number violation and Majorana neutrinos. In fact mechanisms with Dirac neutrino masses might be an indication for physics beyond four dimensions [55].

The most popular way to induce Weinberg's dimension-5 operator is through the so-called seesaw mechanism, which represents a huge variety of possible schemes. The first case is called pure Type-I while the second is pure Type-II, a terminology opposite of the original one suggested in [14], see Fig. 2. Note that the Type-I seesaw mechanism corresponds to having the neutrino mass induced from fermion exchange. In this case, as we will see, neutrino oscillations are effectively described by a non-unitary lepton mixing matrix.

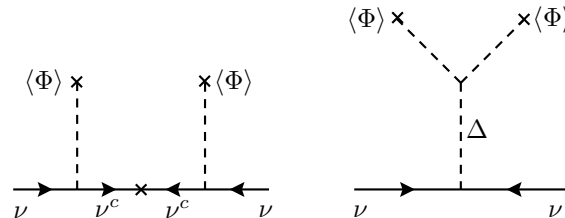


Figure 2: In the Type-I seesaw the neutrino mass is induced from the fermion exchange, while Type-II corresponds to scalar boson exchange.

Given the arbitrariness in the number and transformation properties of extra fermions (for example, the number of right-handed neutrinos is arbitrary, since they carry no anomaly [14]) the seesaw mechanism can be realized at low scale [56, 57, 58, 59, 60]. In this case the messenger particles may be indirectly probed through rare lepton flavour violation decay processes [61, 62, 63, 64, 65, 66] and electroweak precision physics [67, 68, 69], or be directly produced at collider experiments [70, 71].

Before closing this section, let us mention that seesaw extensions of the standard model may have deep implications for new physics, ranging from neutrino physics *stricto sensu* to other aspects of particle physics and cosmology. In particular, the presence of new scalars required in order to break lepton number may affect the stability of the electroweak breaking [72, 73]. Moreover, extra scalars can also induce new contributions to “visible” standard model Higgs decays, such as the $h \rightarrow \gamma\gamma$ and possibly account for new hints, such as the recent diphoton anomaly [74, 75]. There may also be novel Higgs decay channels involving the emission of the Nambu-Goldstone boson associated to spontaneous lepton number violation and neutrino mass generation [76, 77, 78].

4. Non-unitary lepton mixing and seesaw mechanism

The two-component right-handed neutrinos are singlets under the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ symmetry and hence can acquire potentially large gauge invariant masses, breaking total lepton number symmetry. This opens the possibility of generating light neutrino Majorana masses through the exchange of heavy right-handed neutrinos, the so-called Type-I seesaw mechanism. This implies that, in addition to the presence of Majorana phases, the lepton mixing matrix will also couple sub-dominantly the heavy states, leading to a rectangular form for the “PMNS” matrix [14]. These couplings enable the production of the right-handed neutrinos by the charged current weak interaction if the kinematics allows, possibly at the LHC. In most other cases, however, such as oscillations, the heavy states can not participate due to kinematics. As a result, the charged current weak interactions of the light (mass-eigenstate) neutrinos are described by an effective non-unitary mixing matrix.

In order to find the most convenient parametrization for the matrix N describing non-unitary neutrino propagation we start from the unitary matrix $U^{n \times n}$ describing the neutrino diagonalization matrix. In the symmetric parametrization the products of the complex matrices ω_{ij} in Eq. (2) can be chosen in the most convenient way as

$$U^{n \times n} = \omega_{n-1\ n} \omega_{n-2\ n} \dots \omega_{1\ n} \omega_{n-2\ n-1} \omega_{n-3\ n-1} \dots \omega_{1\ n-1} \dots \omega_{2\ 3} \omega_{1\ 3} \omega_{1\ 2}. \quad (7)$$

Following the notation in [79] we break up this matrix $U^{n \times n}$ as

$$U^{n \times n} = \begin{pmatrix} N & S \\ V & T \end{pmatrix}, \quad (8)$$

where N is a 3×3 matrix in the light neutrino sector and S describes the couplings of the extra $n - 3$ isosinglet states, expected to be heavy. The matrix $U^{n \times n}$ can be neatly expanded in perturbation theory [80]. With the ordering shown in Eq. (7) the submatrix N can be decomposed as

$$N = N^{NP} U = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} U, \quad (9)$$

with U the usual unitary 3×3 leptonic mixing matrix. The latter may be expressed as prescribed by the Particle Data Group [81] or in our fully symmetric description, particularly useful for our analyses [15], namely,

$$U = \omega_{2\ 3} \omega_{1\ 3} \omega_{1\ 2}. \quad (10)$$

Note that Eq. (9) gives the most general and convenient description of the lepton mixing matrix relaxing the unitarity approximation, and holds in any seesaw scheme. Notice that in this factorized form the left pre-factor matrix, N^{NP} , characterizes the unitarity violation. Notice that the oscillations of the electron and muon neutrino flavors are described by just four extra parameters, the two real diagonal entries α_{11} and α_{22} plus the complex off-diagonal parameter α_{21} which contains a single additional effective CP phase parameter. In other words only one phase combination enters the “relevant” neutrino oscillation experiments (next section). This important property holds irrespective of the number of extra heavy leptons present. Hence, by conveniently choosing the product ordering of the complex rotation matrices, we obtained a parametrization that concentrates all the information relative to the additional neutral heavy leptons in a compact and simple matrix that contains three zeroes.

Notice also that, because of the chiral nature of the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ model, the couplings of the n neutrino states in the charged current weak interaction will form a rectangular matrix [14], that is,

$$K = \begin{pmatrix} N & S \end{pmatrix}. \quad (11)$$

where the first block corresponds to the three active neutrinos and the second is associated to the other states. The unitarity condition will be replaced by the relation

$$KK^\dagger = NN^\dagger + SS^\dagger = I, \quad (12)$$

with

$$NN^\dagger = \begin{pmatrix} \alpha_{11}^2 & \alpha_{11}\alpha_{21}^* & \alpha_{11}\alpha_{31}^* \\ \alpha_{11}\alpha_{21} & \alpha_{22}^2 + |\alpha_{21}|^2 & \alpha_{22}\alpha_{32}^* + \alpha_{21}\alpha_{31}^* \\ \alpha_{11}\alpha_{31} & \alpha_{22}\alpha_{32} + \alpha_{31}\alpha_{21}^* & \alpha_{33}^2 + |\alpha_{31}|^2 + |\alpha_{32}|^2 \end{pmatrix}. \quad (13)$$

Besides being described by a triangular matrix, another advantage of the parametrization is that the large number of mixing angles and phases coming from any number of extra heavy isosinglets can be reconstructed by relatively simple formulas. This is particularly true for the diagonal elements, α_{ii} , which are expressed as

$$\begin{aligned} \alpha_{11} &= c_{1n} c_{1n-1} c_{1n-2} \dots c_{14}, \\ \alpha_{22} &= c_{2n} c_{2n-1} c_{2n-2} \dots c_{24}, \\ \alpha_{33} &= c_{3n} c_{3n-1} c_{3n-2} \dots c_{34}, \end{aligned} \quad (14)$$

where $c_{ij} = \cos \theta_{ij}$, the cosines of the mixing parameters are real. For the off-diagonal terms, α_{21} and α_{32} , there is also a general and simple formula, given as the sum of $n - 3$ terms

$$\begin{aligned} \alpha_{21} &= c_{2n} c_{2n-1} \dots c_{25} \eta_{24} \bar{\eta}_{14} + c_{2n} \dots c_{26} \eta_{25} \bar{\eta}_{15} c_{14} + \dots + \eta_{2n} \bar{\eta}_{1n} c_{1n-1} c_{1n-2} \dots c_{14}, \\ \alpha_{32} &= c_{3n} c_{3n-1} \dots c_{35} \eta_{34} \bar{\eta}_{24} + c_{3n} \dots c_{36} \eta_{35} \bar{\eta}_{25} c_{24} + \dots + \eta_{3n} \bar{\eta}_{2n} c_{2n-1} c_{2n-2} \dots c_{24}. \end{aligned} \quad (15)$$

These parameters will be complex, and the CP phase information will be encoded in the parameters $\eta_{ij} = e^{-i\phi_{ij}} \sin \theta_{ij}$ and its conjugate $\bar{\eta}_{ij} = -e^{i\phi_{ij}} \sin \theta_{ij}$. Finally, for the α_{31} case, we can neglect quartic terms in $\sin \theta_{ij}$, with $j = 4, 5, \dots$ and find the expression

$$\alpha_{31} = c_{3n} c_{3n-1} \dots c_{35} \eta_{34} c_{24} \bar{\eta}_{14} + c_{3n} \dots c_{36} \eta_{35} c_{25} \bar{\eta}_{15} c_{14} + \dots + \eta_{3n} c_{2n} \bar{\eta}_{1n} c_{1n-1} c_{1n-2} \dots c_{14}. \quad (16)$$

In order to generate a realistic neutrino spectrum at the tree level the Type-I seesaw mechanism requires two right-handed neutrinos. However in the presence of extra Higgs bosons (such as triplets present in Type II seesaw) this is not required. For the simple case of a single extra right-handed neutrino the above expressions give

$$\begin{aligned} \alpha_{11} &= c_{14}, & \alpha_{21} &= \eta_{24} \bar{\eta}_{14}, \\ \alpha_{22} &= c_{24}, & \alpha_{32} &= \eta_{34} \bar{\eta}_{24}, \\ \alpha_{33} &= c_{34}, & \alpha_{31} &= \eta_{34} c_{24} \bar{\eta}_{14}. \end{aligned} \quad (17)$$

Specific expressions for various other interesting seesaw schemes with 3 and 6 heavy leptons are also given in Ref. [82]. The couplings of the heavy right-handed neutrinos allow them to be searched for in many experiments. Moreover their presence implies the effective non-unitarity of the light neutrino mixing matrix, modifying several standard model observables (see Sec. 5) as well as oscillation probabilities which have a very simple form in vacuo as seen in Sec. 6.

5. Current constraints on right-handed neutrinos

Even though right-handed neutrinos as messengers of neutrino mass generation, as postulated in seesaw type schemes, are expected to be heavy, above the weak scale or so, they have been searched for in a variety of situations, starting from much lower masses. Here we start by briefly update the constraints on right-handed neutrinos.

5.1. Low-mass searches

If their mass is low enough, the right-handed states would behave as light sterile neutrinos and would show up at low energies. Indeed, they have been searched for in a variety of weak processes such as pion and kaon weak decay as well as at the LEP experiments [83, 84]. The constraints from direct production of neutral heavy leptons are

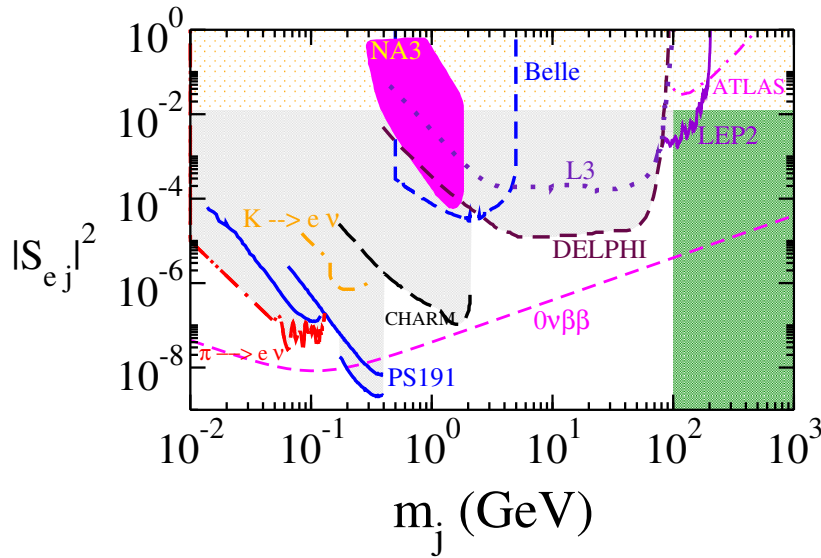


Figure 3: Bounds on the charged current coupling strength of a heavy isosinglet lepton of mass m_j with the electron neutrino. The green vertical band indicates the lower part of the mass region favoured on theoretical grounds, while the horizontal band shows the bound from weak universality.

summarized in Fig. (3), Fig. (4) and Fig. (5). These model-independent limits do not require any particular neutrino mass generation scheme. To obtain these limits, experiments have looked for a resonance in a specific energy window, for a given mixing of the additional heavy state. In Fig. (3), we summarize the constraints on $|S_{ej}|^2$ for a mass range from 10^{-2} to 10^2 GeV coming from the experiments TRIUMF [85, 86] (denoted as $\pi \rightarrow e\nu$ and $K \rightarrow e\nu$ in the plot), PS191 [87], NA3 [88], CHARM [89], Belle [90], the LEP experiments DELPHI [91], L3 [92], LEP2 [93], and the recent LHC results from ATLAS [94, 95]. Future experimental proposals, such as DUNE [96, 97] and ILC, expect to improve these constraints [71].

In Fig. (4) we show the constraints for the mixing of a neutral heavy lepton with a muon neutrino. The bounds from experiments PS191, NA3, Belle, the LEP experiments L3, DELPHI, and the LHC experiment ATLAS, are shown again for this channel. For the muon-type neutrino, there are also bounds from KEK [98, 99] (labeled $K \rightarrow \mu\nu$ in the

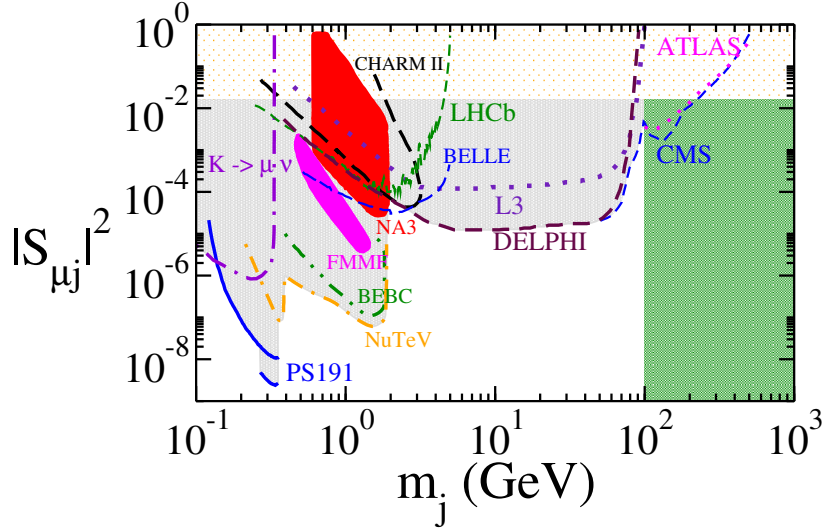


Figure 4: Bounds on the charged current coupling strength of a heavy isosinglet lepton of mass m_j with the muon neutrino. The green vertical band indicates the lower part of the mass region favoured on theoretical grounds, while the horizontal band shows the bound from weak universality.

plot), CHARM II [100], FMMF [101], BEBC [102], NuTeV [103], E949 [104], and the recent constraints from the LHC experiments CMS [105] and LHCb [106]. Finally, we show in Fig. (5) the limits for the charged current coupling strength of a neutral heavy lepton with a tau neutrino. One can see that, as expected, the tau neutrino sector is more difficult to probe so the main constraints come from NOMAD [107], CHARM [108], and DELPHI [91]. Regarding the universality constraint it is obtained from Eq. (23) and implies a forbidden horizontal band in Fig. 5, about 3% at 90% C.L. One should note, however, that the limits for the mixing typically rely upon extra assumptions on the decay

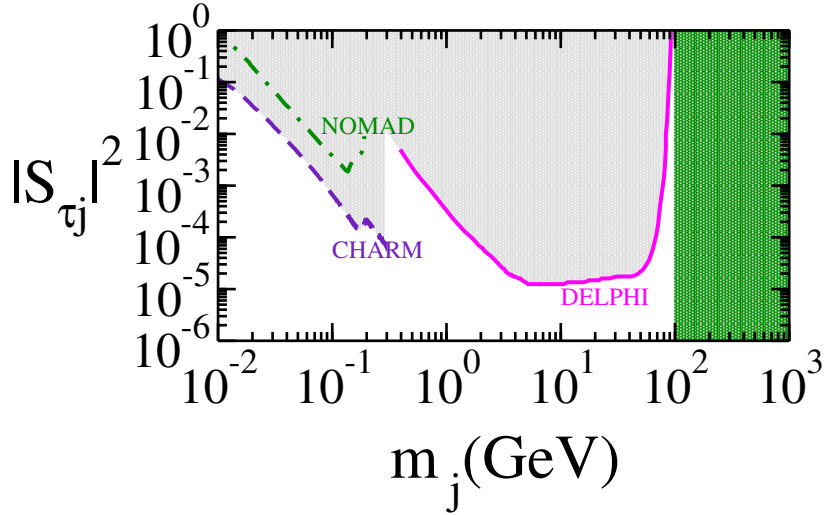


Figure 5: Bounds on the charged current coupling strength of a heavy isosinglet lepton of mass m_j with the tau neutrino. The green vertical band shows the lower part of the mass region favoured on theoretical grounds.

of the neutral heavy lepton, hence somewhat model-dependent. As will be seen below in section 5.2, the existence of neutral heavy leptons affects low energy weak decay processes, where the neutrinos that can be kinematically produced are only the light ones. From those searches one obtains model-independent constraints which are also

shown in Figs. (3) and (4) as a light horizontal region on the top of the figures.

5.2. Weak universality bounds

If right-handed neutrinos are the messengers of neutrino mass generation the corresponding mass eigenstates (also called neutral heavy leptons) will not be emitted in several weak decays, such as beta or muon decays, due to kinematics. Therefore, such decays would measure different values for the Fermi constant, violating universality. In particular, for the aforementioned beta and muon decay, the Fermi constant would be modified to be

$$G_\beta = G_F \sqrt{(NN^\dagger)_{11}} = G_F \sqrt{\alpha_{11}^2} \quad (18)$$

and

$$G_\mu = G_F \sqrt{(NN^\dagger)_{11}(NN^\dagger)_{22}} = G_F \sqrt{\alpha_{11}^2(\alpha_{22}^2 + |\alpha_{21}|^2)} \quad (19)$$

respectively. Since the Fermi coupling constant appears basically in every weak process, almost any observable should be affected by non-unitarity, particularly the well measured CKM matrix elements. Even the pion decay branching ratio

$$R_\pi = \frac{\Gamma(\pi^+ \rightarrow e^+ \nu)}{\Gamma(\pi^+ \rightarrow \mu^+ \nu)} \quad (20)$$

will also be modified by universality violation:

$$r_\pi = \frac{R_\pi}{R_\pi^{SM}} = \frac{(NN^\dagger)_{11}}{(NN^\dagger)_{22}} = \frac{\alpha_{11}^2}{\alpha_{22}^2 + |\alpha_{21}|^2}. \quad (21)$$

Universality constraints derived from the CKM matrix elements [109] as well as from pion decay [110] has been extensively analyzed and dedicated studies have been devoted to this subject [111, 112, 113, 114, 115]. Using the above prescriptions for the Fermi constants, and based on the experimental measurements for the CKM matrix elements [81] and for the pion branching ratio [116] one obtains, at 90% C.L.,

$$\begin{aligned} 1 - \alpha_{11}^2 &< 0.0130, \\ 1 - \alpha_{22}^2 - |\alpha_{21}|^2 &< 0.0012. \end{aligned} \quad (22)$$

For the case of $\mu - \tau$ universality test, one can use the few experimental data available [117] in order to get a constraint

$$\frac{(NN^\dagger)_{33}}{(NN^\dagger)_{22}} = 0.9850 \pm 0.0057. \quad (23)$$

5.3. High energy coliders

More interesting is the possibility that right-handed neutrinos are the messengers of neutrino mass generation. In this case the smallness of neutrino masses may severely restrict the magnitudes of the right-handed neutrino admixtures and hence close their potential signatures at collider experiments like the LHC. This is expected in the high-scale Type-I seesaw. However, these limitations can be avoided within a broad class of low-scale seesaw realizations, such as the inverse seesaw [56, 57, 58, 59, 60] and linear seesaw [118, 119, 120, 121] mechanisms.

Within the standard $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ model heavy neutrinos in the TeV range, would be produced only through relatively small mixing effects, since they are mainly isosinglets. Still signatures associated with such heavy neutrinos can be searched for directly at accelerator experiments, like LEP and the LHC. Indeed, as can be seen from Figs. 3 and 4 the restrictions from the LHC are weaker than what would be expected from unitarity violation bounds in this mass range. LHC constraints are currently absent in the case of the tau-flavor, as seen in Fig. 5, though in principle one expects that future LHC runs will improve the situation [122].

In contrast, the above limitation can be avoided in extended electroweak models with larger gauge groups. In such case the extra kinematically accessible gauge bosons provide a production portal for the heavy neutrinos which may be copiously produced, and can also give rise to lepton flavour violation signatures. As an example one can have left-right symmetric models, which lead to processes with lepton flavour violation at high energies [123, 70]. Another interesting extension are models with $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ gauge symmetry [124, 125, 126, 127], which have many interesting features as well as experimental signatures.

5.4. Neutrinoless double beta decay

Since current knowledge of the neutrino mixing matrix comes exclusively from oscillation experiments, there is no information on the absolute neutrino mass scale, neither on the Majorana phases. A possible detection of neutrinoless double beta decay would imply that neutrinos are their own anti-particles [128] irrespective of the underlying origin of neutrino mass, as illustrated in Fig. 6. It would also provide complementary information inaccessible within oscillation

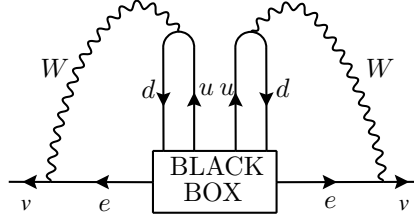


Figure 6: Neutrinoless double beta decay implies that neutrinos are own anti-particles [128].

studies and the searches described above [129, 130]. The effective Majorana neutrino mass will be given by [129]

$$\langle m \rangle = \left| \sum_j (U_{ej}^{n \times n})^2 m_j \right|. \quad (24)$$

Notice that j runs only for the light neutrinos. The presence of the heavy neutrinos modifies the mixing matrix entries as: $U_{ej}^{n \times n} = \alpha_{11} U_{ej}$. Moreover, heavy neutrino exchange will lead short-range $0\nu\beta\beta$ contributions. These will be suppressed due to their mixing in the rectangular charged current mixing matrix, and will involve a different mass dependence, since the $0\nu\beta\beta$ amplitude is proportional to

$$\mathcal{A} \propto \frac{m_j}{q^2 - m_j^2}. \quad (25)$$

For neutrino masses above the typical neutrino momentum (around 0.1 GeV), the amplitude will be inversely proportional to the mass of the heavy state. We show in Fig. (3) the limit for S_{ej} obtained from ^{76}Ge for a single massive isosinglet neutrino [131, 132]. We stress that this limit holds only if neutrinos have Majorana nature, a restriction that is not required for the other constraints shown on the same figure.

6. Future tests of non-unitarity in neutrino oscillations

As discussed above, the presence of extra heavy leptons modifies the standard form of the leptonic mixing matrix. For example, leptonic mixing, as well as CP violation, may take place even in the limit where neutrinos become strictly massless [133, 134]. This leads to the possibility of lepton flavour violation and CP violating processes [61, 135]. Moreover, it leads to new conceptual possibilities for neutrino propagation which could have dramatic implications in astrophysical environments [133, 136, 137].

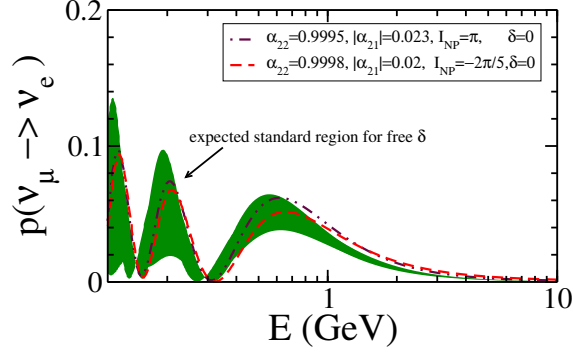


Figure 7: Conversion probability for a fixed baseline of 295 km. The green region shows the standard conversion probability with all the parameters fixed to the current best fit value, except for the CP phase $\delta = -I_{123}$ which is left free. The non-unitary case is illustrated with vanishing standard CP phase and two different choices of the new physics parameters: as indicated.

Non-standard properties such as unitarity violation in neutrino mixing could also be probed in laboratory studies. Indeed the survival and conversion neutrino oscillation probabilities should be modified to

$$P_{\alpha\beta} = \sum_{i,j}^3 N_{\alpha i}^* N_{\beta i} N_{\alpha j} N_{\beta j}^* - 4 \sum_{j>i}^3 \text{Re} [N_{\alpha j}^* N_{\beta j} N_{\alpha i} N_{\beta i}^*] \sin^2 \left(\frac{\Delta m_{ji}^2 L}{4E} \right) + 2 \sum_{j>i}^3 \text{Im} [N_{\alpha j}^* N_{\beta j} N_{\alpha i} N_{\beta i}^*] \sin \left(\frac{\Delta m_{ji}^2 L}{2E} \right). \quad (26)$$

For instance, for the case of muon to electron neutrino conversion the transition probability, in vacuum, will be given by the very simple approximate formula [82]

$$P_{\mu e} = \alpha_{11}^2 |\alpha_{21}|^2 + (\alpha_{11} \alpha_{22})^2 P_{\mu e}^{3 \times 3} + \alpha_{11}^2 \alpha_{22} |\alpha_{21}| P_{\mu e}^I. \quad (27)$$

Here, $P_{\mu e}^{3 \times 3}$ is the standard three-neutrino oscillation formula [138]. Notice that in this case a new constant term appears, $\alpha_{22}^2 |\alpha_{21}|^2$, that accounts for the effect of non-unitarity at zero distance, associated to the effective non-orthogonality of the weak eigenstate neutrinos [133]. Finally, the new term, $P_{\mu e}^I$, will depend on two different phases: the standard CP phase δ characterizing three-neutrino oscillations, and an additional CP phase associated to the new physics, I_{NP} , given by the argument of the complex parameter $\alpha_{21} = |\alpha_{21}| \exp(I_{NP})$:

$$P_{\mu e}^I = -2 \left[\sin(2\theta_{13}) \sin \theta_{23} \sin \left(\frac{\Delta m_{31}^2 L}{4E_\nu} \right) \sin \left(\frac{\Delta m_{31}^2 L}{4E_\nu} + I_{NP} - I_{123} \right) \right] - \cos \theta_{13} \cos \theta_{23} \sin(2\theta_{12}) \sin \left(\frac{\Delta m_{21}^2 L}{2E_\nu} \right) \sin(I_{NP}). \quad (28)$$

Therefore, the effect of non-unitarity can be described by four real parameters : α_{11} , α_{22} , $|\alpha_{21}|$ plus the phase I_{NP} .

The conversion probability for the neutrino appearance channel in the T2K experiment, characterized by a 295 km baseline [18, 19], is given in Fig. (7). The green region shows the standard conversion probability with all the oscillation parameters fixed to their current best fit value, except for the new CP phase which has been left free. On the other hand the non-unitary case is illustrated with a standard CP phase equal to zero and two different choices of the new physics parameters: $\alpha_{11} = 1$, $\alpha_{22} = 0.9995$, $|\alpha_{21}| = 0.023$, and $I_{NP} = \pi$ (maroon dashed-dotted line); and $\alpha_{11} = 1$, $\alpha_{22} = 0.9998$, $|\alpha_{21}| = 0.02$, and $I_{NP} = -2\pi/5$ (red dashed line).

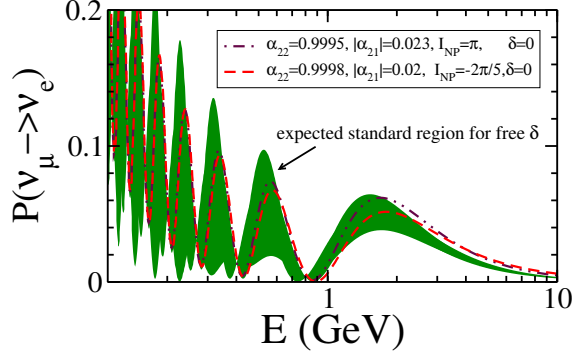


Figure 8: Conversion probability for a fixed baseline of 810 km. The green region shows the standard conversion probability with all the parameters fixed to the current best fit value, except for the CP phase $\delta = -I_{123}$ which is left free. The non-unitary case is illustrated with a vanishing standard CP phase and two different choices of the new physics parameters, as indicated.

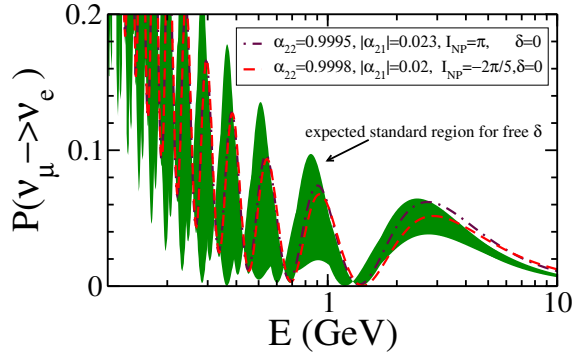


Figure 9: Conversion probability for a fixed baseline of 1,300 km. The green region shows the standard conversion probability with all the parameters fixed to the current best fit value, except for the CP phase $\delta = -I_{123}$ that has been left free. The non-unitary case is illustrated with a standard CP phase equal to zero and two different choices of the new physics parameters, as indicated.

Recently the NOvA experiment that has reported a new measurement of the electron neutrino appearance channel [139]. Motivated by this we show, in Fig (8), the behaviour of the conversion probability for the case of a 810 km baseline. The green region shows the standard conversion probability with all the parameters fixed to the current best fit value, except for the new CP phase $\delta = -I_{123}$, which has been left free. The non-unitary case is illustrated with a standard CP phase equal to zero and two different choices of the new physics parameters, indicated in the plot.

Finally, in Fig. (9) we plot the conversion probability for a baseline of 1,300 km, relevant for the future DUNE proposal [97]. Although we have neglected matter effects, our results illustrate pretty well the main qualitative point. The shaded band in this figure is the standard region for the central values of the current neutrino oscillation parameters as reported in Ref. [11], leaving the CP phase completely free. The panels also show two survival probabilities including non-unitarity effects, for two particular choices of parameters. In these two new physics cases we have set the standard CP phase to zero and we have taken a non-zero value for the extra phase I_{NP} . One sees that, in all the above cases, our results are suggestive of the fact that there is room for a degeneracies between standard oscillations and new physics associated to non-unitarity or “seesaw” effects. These will make it difficult to extract the standard CP effects and will certainly be one of the challenges which future neutrino experiments will have to face.

7. Conclusions

Here we gave a brief summary of the theoretical interpretation of current neutrino oscillation data within the three-neutrino paradigm. If neutrinos get their mass *a la seesaw*, we may expect both direct and/or indirect effects associated to the neutrino mass generation messengers, e.g. the heavy right-handed neutrinos. These could be indicative of the simplest next step in particle physics. Insofar as oscillations are concerned, we pointed out the case for unitarity violation in the “PMNS” matrix. Experiments are usually interpreted within the unitary approximation. However, we illustrated how CP violation studies could confuse genuine CP violation with effects associated with unitarity deviations. Taking up this challenge would shed light on the mass scale of lepton number violation and neutrino mass generation within the seesaw mechanism. Besides opening the stage for new physics, refined neutrino oscillation studies might also pave the way towards the understanding of at least some of the current puzzles facing modern cosmology. The reader is addressed to [140, 141, 142] for examples of possible cosmological implications of neutrino mass generation.

The interpretation of neutrino data has also been considered in terms of sub-Fermi strength non-standard interactions of a more generic type than the non-unitarity effects considered above [143, 144, 145, 146]. Laboratory oscillation studies not only give crucial confirmation of the oscillation hypothesis, but also establish the robustness of large mixing angle solar neutrino oscillations [52, 53]. One exception is the existence of a large mixing solution, in the dark side, which still survives [147, 148]. Here we have focussed on the simplest manifestation of non-standard neutrino propagation, namely that which comes from the non-unitary form of the lepton mixing matrix and characterizes Type I seesaw schemes.

Acknowledgements

This work is supported by the Spanish grants FPA2014-58183-P, Multidark CSD2009-00064, SEV-2014-0398 (MINECO), PROMETEOII/2014/084 (Generalitat Valenciana), and the CONACyT grant 166639.

References

- [1] G. Aad, et al., Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, Phys.Lett. B716 (2012) 1–29. arXiv:1207.7214, doi:10.1016/j.physletb.2012.08.020.
- [2] Y. Fukuda, et al., Evidence for oscillation of atmospheric neutrinos, Phys. Rev. Lett. 81 (1998) 1562–1567. arXiv:hep-ex/9807003.
- [3] Y. Ashie, et al., Evidence for an oscillatory signature in atmospheric neutrino oscillation, Phys. Rev. Lett. 93 (2004) 101801. arXiv:hep-ex/0404034, doi:10.1103/PhysRevLett.93.101801.
- [4] S. Fukuda, et al., Determination of solar neutrino oscillation parameters using 1496 days of Super-Kamiokande-I data, Phys. Lett. B539 (2002) 179–187. arXiv:hep-ex/0205075.
- [5] Q. R. Ahmad, et al., Direct evidence for neutrino flavor transformation from neutral-current interactions in the Sudbury Neutrino Observatory, Phys. Rev. Lett. 89 (2002) 011301. arXiv:nucl-ex/0204008.
- [6] K. Eguchi, et al., First results from KamLAND: Evidence for reactor anti- neutrino disappearance, Phys. Rev. Lett. 90 (2003) 021802. arXiv:hep-ex/0212021.
- [7] H. Georgi, S. Glashow, Unity of All Elementary Particle Forces, Phys.Rev.Lett. 32 (1974) 438–441.
- [8] H. Georgi, H. R. Quinn, S. Weinberg, Hierarchy of interactions in unified gauge theories, Phys. Rev. Lett. 33 (1974) 451–454.
- [9] A. Djouadi, The Anatomy of electro-weak symmetry breaking. I: The Higgs boson in the standard model, Phys.Rept. 457 (2008) 1–216.
- [10] M. Maltoni, T. Schwetz, M. Tortola, J. Valle, Status of global fits to neutrino oscillations, New J.Phys. 6 (2004) 122. arXiv:hep-ph/0405172, doi:10.1088/1367-2630/6/1/122.
- [11] D. Forero, M. Tortola, J. Valle, Neutrino oscillations refitted, Phys.Rev. D90 (9) (2014) 093006. arXiv:1405.7540, doi:10.1103/PhysRevD.90.093006.
- [12] J. W. Valle, J. C. Romao, Neutrinos in high energy and astroparticle physics, 1st Edition, Wiley-VCH, Berlin, 2015.
- [13] M. Kobayashi, T. Maskawa, CP violation in the renormalizable theory of weak interaction, Prog. Theor. Phys. 49 (1973) 652–657.
- [14] J. Schechter, J. Valle, Neutrino Masses in SU(2) x U(1) Theories, Phys.Rev. D22 (1980) 2227. doi:10.1103/PhysRevD.22.2227.
- [15] W. Rodejohann, J. W. F. Valle, Symmetrical Parametrizations of the Lepton Mixing Matrix, Phys.Rev. D84 (2011) 073011. arXiv:1108.3484.

- [16] J. Schechter, J. W. F. Valle, Neutrino oscillation thought experiment, *Phys. Rev. D* 23 (1981) 1666.
- [17] M. Doi, T. Kotani, H. Nishiura, K. Okuda, E. Takasugi, CP Violation in Majorana Neutrinos, *Phys. Lett. B* 102 (1981) 323.
- [18] K. Abe, et al., Indication of Electron Neutrino Appearance from an Accelerator-produced Off-axis Muon Neutrino Beam, *Phys.Rev.Lett.* 107 (2011) 041801. arXiv:1106.2822.
- [19] K. Abe, et al., Observation of Electron Neutrino Appearance in a Muon Neutrino Beam, *Phys. Rev. Lett.* 112 (2014) 061802. arXiv:1311.4750, doi:10.1103/PhysRevLett.112.061802.
- [20] P. Adamson, et al., Improved search for muon-neutrino to electron-neutrino oscillations in MINOS, *Phys.Rev.Lett.* 107 (2011) 181802. arXiv:1108.0015.
- [21] P. Adamson, et al., Measurement of Neutrino and Antineutrino Oscillations Using Beam and Atmospheric Data in MINOS, *Phys.Rev.Lett.* 110 (2013) 251801. arXiv:1304.6335.
- [22] P. Adamson, et al., Electron neutrino and antineutrino appearance in the full MINOS data sample, *Phys. Rev. Lett.* 110 (17) (2013) 171801. arXiv:1301.4581, doi:10.1103/PhysRevLett.110.171801.
- [23] Y. Abe, et al., Indication for the disappearance of reactor electron antineutrinos in the Double Chooz experiment, *Phys.Rev.Lett.* 108 (2012) 131801. arXiv:1112.6353.
- [24] J. Ahn, et al., Observation of Reactor Electron Antineutrino Disappearance in the RENO Experiment, *Phys.Rev.Lett.* 108 (2012) 191802. arXiv:1204.0626.
- [25] F. An, et al., Observation of electron-antineutrino disappearance at Daya Bay, *Phys.Rev.Lett.* 108 (2012) 171803. arXiv:1203.1669.
- [26] B. Aharmim, et al., Low Energy Threshold Analysis of the Phase I and Phase II Data Sets of the Sudbury Neutrino Observatory, *Phys. Rev. C* 81 (2010) 055504. arXiv:0910.2984.
- [27] B. Aharmim, et al., An Independent Measurement of the Total Active 8B Solar Neutrino Flux Using an Array of 3He Proportional Counters at the Sudbury Neutrino Observatory, *Phys. Rev. Lett.* 101 (2008) 111301. arXiv:0806.0989.
- [28] K. Abe, et al., Solar neutrino results in Super-Kamiokande-III, *Phys.Rev. D* 83 (2011) 052010. arXiv:1010.0118.
- [29] A. Renshaw, Solar Neutrino Results from Super-Kamiokande, *Phys. Procedia* 61 (2015) 345–354. arXiv:1403.4575, doi:10.1016/j.phpro.2014.12.074.
- [30] J. Hosaka, et al., Solar neutrino measurements in Super-Kamiokande-I, *Phys. Rev. D* 73 (2006) 112001. arXiv:hep-ex/0508053.
- [31] J. Cravens, et al., Solar neutrino measurements in Super-Kamiokande-II, *Phys.Rev. D* 78 (2008) 032002. arXiv:0803.4312.
- [32] G. Bellini, J. Benziger, D. Bick, S. Bonetti, G. Bonfini, et al., Precision measurement of the ^7Be solar neutrino interaction rate in Borexino, *Phys.Rev.Lett.* 107 (2011) 141302. arXiv:1104.1816.
- [33] J. N. Abdurashitov, et al., Measurement of the solar neutrino capture rate with gallium metal. III: Results for the 2002–2007 data-taking period, *Phys. Rev. C* 80 (2009) 015807. arXiv:0901.2200.
- [34] F. Kaether, W. Hampel, G. Heusser, J. Kiko, T. Kirsten, Reanalysis of the GALLEX solar neutrino flux and source experiments, *Phys.Lett. B* 685 (2010) 47–54. arXiv:1001.2731.
- [35] B. T. Cleveland, et al., Measurement of the solar electron neutrino flux with the homestake chlorine detector, *Astrophys. J.* 496 (1998) 505–526.
- [36] R. Wendell, et al., Atmospheric neutrino oscillation analysis with sub-leading effects in Super-Kamiokande I, II, and III, *Phys.Rev. D* 81 (2010) 092004. arXiv:1002.3471.
- [37] F. Capozzi, G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, Status of three-neutrino oscillation parameters, circa 2013, *Phys. Rev. D* 89 (2014) 093018. arXiv:1312.2878, doi:10.1103/PhysRevD.89.093018.
- [38] M. C. Gonzalez-Garcia, M. Maltoni, T. Schwetz, Updated fit to three neutrino mixing: status of leptonic CP violation, *JHEP* 11 (2014) 052. arXiv:1409.5439, doi:10.1007/JHEP11(2014)052.
- [39] W. Haxton, R. Hamish Robertson, A. M. Serenelli, Solar Neutrinos: Status and Prospects, *Ann. Rev. Astron. Astrophys.* 51 (2013) 21–61. arXiv:1208.5723.
- [40] A. Balantekin, W. Haxton, Neutrino Oscillations, *Prog.Part.Nucl.Phys.* 71 (2013) 150–161. arXiv:1303.2272.
- [41] A. B. Balantekin, J. M. Fetter, F. N. Loreti, The MSW effect in a fluctuating matter density, *Phys. Rev. D* 54 (1996) 3941–3951. arXiv:astro-ph/9604061.
- [42] H. Nunokawa, et al., The effect of random matter density perturbations on the MSW solution to the solar neutrino problem, *Nucl. Phys. B* 472 (1996) 495–517. arXiv:hep-ph/9602307.
- [43] C. P. Burgess, et al., Resonant origin for density fluctuations deep within the sun: helioseismology and magneto-gravity waves, *Mon. Not. Roy. Astron. Soc.* 348 (2004) 609. arXiv:astro-ph/0304462.
- [44] V. Castellani, S. Degl’Innocenti, W. A. Dziembowski, G. Fiorentini, B. Ricci, Helioseismology, solar models and neutrino fluxes, *Nucl. Phys. Proc. Suppl.* 70 (1999) 301–314. arXiv:astro-ph/9712174.
- [45] J. Christensen-Dalsgaard, Helioseismology, *Rev. Mod. Phys.* 74 (2003) 1073–1129. arXiv:astro-ph/0207403.
- [46] C. Burgess, et al., Large mixing angle oscillations as a probe of the deep solar interior, *Astrophys. J.* 588 (2003) L65. arXiv:hep-ph/0209094.
- [47] C. P. Burgess, et al., Cornering solar radiative-zone fluctuations with KamLAND and SNO salt, *JCAP* 0401 (2004) 007.
- [48] O. G. Miranda, et al., The simplest resonant spin-flavour solution to the solar neutrino problem, *Nucl. Phys. B* 595 (2001) 360–380.
- [49] J. Barranco, et al., Confronting spin flavor solutions of the solar neutrino problem with current and future solar neutrino data, *Phys. Rev. D* 66 (2002) 093009. arXiv:hep-ph/0207326 v3 KamLAND-updated version.

- [50] O. G. Miranda, T. I. Rashba, A. I. Rez, J. W. F. Valle, Enhanced solar anti-neutrino flux in random magnetic fields, *Phys. Rev. D* 70 (2004) 113002. [arXiv:hep-ph/0406066](#).
- [51] O. G. Miranda, et al., Constraining the neutrino magnetic moment with anti- neutrinos from the sun, *Phys. Rev. Lett.* 93 (2004) 051304. [arXiv:hep-ph/0311014](#).
- [52] M. Maltoni, T. Schwetz, J. W. F. Valle, Combining first KamLAND results with solar neutrino data, *Phys. Rev. D* 67 (2003) 093003. [arXiv:hep-ph/0212129](#).
- [53] S. Pakvasa, J. W. F. Valle, Neutrino properties before and after KamLAND *Proc. of the Indian National Acad. of Sci. on Neutrinos*, Vol. 70A, No.1, p.189 - 222 (2004), Eds. D. Indumathi, M. Murthy and G. Rajasekaran. [arXiv:hep-ph/0301061](#).
- [54] A. Aranda, C. Bonilla, S. Morisi, E. Peinado, J. W. F. Valle, Dirac neutrinos from flavor symmetry, *Phys. Rev. D* 89 (3) (2014) 033001. [arXiv:1307.3553](#), doi:10.1103/PhysRevD.89.033001.
- [55] P. Chen, G.-J. Ding, A. D. Rojas, C. A. Vaquera-Araujo, J. W. F. Valle, Warped flavor symmetry predictions for neutrino physics, *JHEP* 01 (2016) 007. [arXiv:1509.06683](#), doi:10.1007/JHEP01(2016)007.
- [56] R. N. Mohapatra, J. W. F. Valle, Neutrino mass and baryon-number nonconservation in superstring models, *Phys. Rev. D* 34 (1986) 1642.
- [57] M. C. Gonzalez-Garcia, J. W. F. Valle, Fast decaying neutrinos and observable flavor violation in a new class of Majoron models, *Phys. Lett. B* 216 (1989) 360.
- [58] E. K. Akhmedov, M. Lindner, E. Schnapka, J. Valle, Left-right symmetry breaking in NJL approach, *Phys.Lett. B* 368 (1996) 270–280. [arXiv:hep-ph/9507275](#), doi:10.1016/0370-2693(95)01504-3.
- [59] E. K. Akhmedov, M. Lindner, E. Schnapka, J. Valle, Dynamical left-right symmetry breaking, *Phys.Rev. D* 53 (1996) 2752–2780. [arXiv:hep-ph/9509255](#), doi:10.1103/PhysRevD.53.2752.
- [60] M. Malinsky, J. Romao, J. Valle, Novel supersymmetric SO(10) seesaw mechanism, *Phys.Rev.Lett.* 95 (2005) 161801. [arXiv:hep-ph/0506296](#), doi:10.1103/PhysRevLett.95.161801.
- [61] J. Bernabeu, et al., Lepton flavor nonconservation at high-energies in a superstring inspired standard model, *Phys. Lett. B* 187 (1987) 303.
- [62] F. F. Deppisch, N. Desai, J. W. F. Valle, Is charged lepton flavour violation a high energy phenomenon?, *Phys.Rev. D* 89 (2014) 051302(R). [arXiv:1308.6789](#).
- [63] F. Deppisch, J. W. F. Valle, Enhanced lepton flavour violation in the supersymmetric inverse seesaw model, *Phys. Rev. D* 72 (2005) 036001. [arXiv:hep-ph/0406040](#).
- [64] F. Deppisch, T. S. Kosmas, J. W. F. Valle, Enhanced μ - e - conversion in nuclei in the inverse seesaw model, *Nucl. Phys. B* 752 (2006) 80–92. [arXiv:hep-ph/0512360](#).
- [65] A. Abada, M. E. Krauss, W. Porod, F. Staub, A. Vicente, C. Weiland, Lepton flavor violation in low-scale seesaw models: SUSY and non-SUSY contributions, *JHEP* 11 (2014) 048. [arXiv:1408.0138](#), doi:10.1007/JHEP11(2014)048.
- [66] S. M. Boucenna, J. W. F. Valle, A. Vicente, Predicting charged lepton flavor violation from 3-3-1 gauge symmetry, *Phys. Rev. D* 92 (5) (2015) 053001. [arXiv:1502.07546](#), doi:10.1103/PhysRevD.92.053001.
- [67] M. E. Peskin, T. Takeuchi, New constraint on a strongly interacting higgs sector, *Phys. Rev. Lett.* 65 (1990) 964–967. doi:10.1103/PhysRevLett.65.964
URL <http://link.aps.org/doi/10.1103/PhysRevLett.65.964>
- [68] G. Altarelli, R. Barbieri, Vacuum polarization effects of new physics on electroweak processes, *Physics Letters B* 253 (1) (1991) 161 – 167. doi:[http://dx.doi.org/10.1016/0370-2693\(91\)91378-9](http://dx.doi.org/10.1016/0370-2693(91)91378-9).
URL <http://www.sciencedirect.com/science/article/pii/0370269391913789>
- [69] M. E. Peskin, T. Takeuchi, Estimation of oblique electroweak corrections, *Phys. Rev. D* 46 (1992) 381–409. doi:10.1103/PhysRevD.46.381. URL <http://link.aps.org/doi/10.1103/PhysRevD.46.381>
- [70] S. Das, F. Deppisch, O. Kittel, J. W. F. Valle, Heavy Neutrinos and Lepton Flavour Violation in Left-Right Symmetric Models at the LHC, *Phys.Rev. D* 86 (2012) 055006. [arXiv:1206.0256](#).
- [71] F. F. Deppisch, P. S. Bhupal Dev, A. Pilaftsis, Neutrinos and Collider Physics, *New J. Phys.* 17 (7) (2015) 075019. [arXiv:1502.06541](#), doi:10.1088/1367-2630/17/7/075019.
- [72] C. Bonilla, R. M. Fonseca, J. W. F. Valle, Vacuum stability with spontaneous violation of lepton number [arXiv:1506.04031](#).
- [73] C. Bonilla, R. M. Fonseca, J. W. F. Valle, Consistency of the triplet seesaw model revisited, *Phys. Rev. D* 92 (7) (2015) 075028.
- [74] S. M. Boucenna, S. Morisi, A. Vicente, The LHC diphoton resonance from gauge symmetry [arXiv:1512.06878](#).
- [75] P. V. Dong, N. T. K. Ngan, Phenomenology of the simple 3-3-1 model with inert scalars [arXiv:1512.09073](#).
- [76] A. S. Joshipura, J. W. F. Valle, Invisible Higgs decays and neutrino physics, *Nucl. Phys. B* 397 (1993) 105–122.
- [77] M. A. Diaz, M. A. Garcia-Jareno, D. A. Restrepo, J. W. F. Valle, Seesaw Majoron model of neutrino mass and novel signals in Higgs boson production at LEP, *Nucl. Phys. B* 527 (1998) 44–60. [arXiv:hep-ph/9803362](#), doi:10.1016/S0550-3213(98)00434-9.
- [78] C. Bonilla, J. W. F. Valle, J. C. Romao, Neutrino mass and invisible Higgs decays at the LHC, *Phys. Rev. D* 91 (11) (2015) 113015. [arXiv:1502.01649](#), doi:10.1103/PhysRevD.91.113015.
- [79] H. Hettmansperger, M. Lindner, W. Rodejohann, Phenomenological Consequences of sub-leading Terms in See-Saw Formulas, *JHEP* 1104 (2011) 123. [arXiv:1102.3432](#).
- [80] J. Schechter, J. W. F. Valle, Neutrino Decay and Spontaneous Violation of Lepton Number, *Phys. Rev. D* 25 (1982) 774.
- [81] K. Olive, et al., Review of Particle Physics, *Chin.Phys.* C38 (2014) 090001. doi:10.1088/1674-1137/38/9/090001.

- [82] F. J. Escrivuela, D. V. Forero, O. G. Miranda, M. Tortola, J. W. F. Valle, On the description of non-unitary neutrino mixing, *Phys. Rev. D* **92** (5) (2015) 053009. arXiv:1503.08879, doi:10.1103/PhysRevD.92.053009.
- [83] M. Dittmar, A. Santamaria, M. C. Gonzalez-Garcia, J. W. F. Valle, Production mechanisms and signatures of isosinglet neutral heavy leptons in Z^0 decays, *Nucl. Phys. B* **332** (1990) 1.
- [84] M. Akrawy, et al., Limits on neutral heavy lepton production from Z^0 decay, *Phys.Lett. B* **247** (1990) 448–457. doi:10.1016/0370-2693(90)90924-U.
- [85] D. Britton, S. Ahmad, D. Bryman, R. Burnham, E. Clifford, et al., Measurement of the $\pi^+ \rightarrow e^+ \nu$ neutrino branching ratio, *Phys.Rev.Lett.* **68** (1992) 3000–3003. doi:10.1103/PhysRevLett.68.3000.
- [86] D. Britton, S. Ahmad, D. Bryman, R. Burnham, E. Clifford, et al., Improved search for massive neutrinos in $\pi^+ \rightarrow e^+ \nu$ neutrino decay, *Phys.Rev. D* **46** (1992) 885–887. doi:10.1103/PhysRevD.46.R885.
- [87] G. Bernardi, G. Carugno, J. Chauveau, F. Dicarilo, M. Dris, et al., Further limits on heavy neutrino couplings, *Phys.Lett. B* **203** (1988) 332. doi:10.1016/0370-2693(88)90563-1.
- [88] J. Badier, et al., Mass and Lifetime Limits on New Longlived Particles in 300-GeV/ $c\pi^-$ Interactions, *Z.Phys. C* **31** (1986) 21. doi:10.1007/BF01559588.
- [89] F. Bergsma, et al., A Search for Decays of Heavy Neutrinos in the Mass Range 0.5-GeV to 2.8-GeV, *Phys.Lett. B* **166** (1986) 473. doi:10.1016/0370-2693(86)91601-1.
- [90] D. Liventsev, et al., Search for heavy neutrinos at Belle, *Phys.Rev. D* **87** (7) (2013) 071102. arXiv:1301.1105, doi:10.1103/PhysRevD.87.071102.
- [91] P. Abreu, et al., Search for neutral heavy leptons produced in Z decays, *Z.Phys. C* **74** (1997) 57–71. doi:10.1007/s002880050370.
- [92] O. Adriani, et al., Search for isosinglet neutral heavy leptons in Z^0 decays, *Phys.Lett. B* **295** (1992) 371–382. doi:10.1016/0370-2693(92)91579-X.
- [93] P. Achard, et al., Search for heavy isosinglet neutrino in e^+e^- annihilation at LEP, *Phys.Lett. B* **517** (2001) 67–74. arXiv:hep-ex/0107014, doi:10.1016/S0370-2693(01)00993-5.
- [94] Search for Majorana neutrino production in pp collisions at $\sqrt{s}=7$ TeV in dimuon final states with the ATLAS detector.
- [95] G. Aad, et al., Search for heavy Majorana neutrinos with the ATLAS detector in pp collisions at $\sqrt{s} = 8$ TeV, *JHEP* **07** (2015) 162. arXiv:1506.06020, doi:10.1007/JHEP07(2015)162.
- [96] C. Adams, et al., The Long-Baseline Neutrino Experiment: Exploring Fundamental Symmetries of the Universe arXiv:1307.7335.
- [97] R. Acciarri, et al., Long-Baseline Neutrino Facility (LBNF) and Deep Underground Neutrino Experiment (DUNE) Conceptual Design Report Volume 2: The Physics Program for DUNE at LBNF arXiv:1512.06148.
- [98] R. Hayano, T. Taniguchi, T. Yamanaka, T. Tanimori, R. Enomoto, et al., Heavy neutrino search using $K(\mu^2)$ decay, *Phys.Rev.Lett.* **49** (1982) 1305. doi:10.1103/PhysRevLett.49.1305.
- [99] A. Kusenko, S. Pascoli, D. Semikoz, New bounds on MeV sterile neutrinos based on the accelerator and Super-Kamiokande results, *JHEP* **0511** (2005) 028. arXiv:hep-ph/0405198, doi:10.1088/1126-6708/2005/11/028.
- [100] P. Vilain, et al., Search for heavy isosinglet neutrinos, *Phys.Lett. B* **343** (1995) 453–458. doi:10.1016/0370-2693(94)01422-9.
- [101] E. Gallas, et al., Search for neutral weakly interacting massive particles in the Fermilab Tevatron wide band neutrino beam, *Phys.Rev. D* **52** (1995) 6–14. doi:10.1103/PhysRevD.52.6.
- [102] A. M. Cooper-Sarkar, et al., Search for Heavy Neutrino Decays in the BEBC Beam Dump Experiment, *Phys.Lett. B* **160** (1985) 207. doi:10.1016/0370-2693(85)91493-5.
- [103] A. Vaitaitis, et al., Search for neutral heavy leptons in a high-energy neutrino beam, *Phys.Rev.Lett.* **83** (1999) 4943–4946. arXiv:hep-ex/9908011, doi:10.1103/PhysRevLett.83.4943.
- [104] A. Artamonov, et al., Search for heavy neutrinos in $K^+ \rightarrow \mu^+ \nu_H$ decays, *Phys.Rev. D* **91** (5) (2015) 052001. arXiv:1411.3963, doi:10.1103/PhysRevD.91.059903, 10.1103/PhysRevD.91.052001.
- [105] V. Khachatryan, et al., Search for heavy Majorana neutrinos in $\mu^+\mu^+$ +jets events in proton-proton collisions at $\sqrt{s} = 8$ TeV arXiv:1501.05566.
- [106] R. Aaij, et al., Search for Majorana neutrinos in $B^- \rightarrow \pi^+ \mu^- \mu^-$ decays, *Phys.Rev.Lett.* **112** (13) (2014) 131802. arXiv:1401.5361, doi:10.1103/PhysRevLett.112.131802.
- [107] P. Astier, et al., Search for heavy neutrinos mixing with tau neutrinos, *Phys.Lett. B* **506** (2001) 27–38. arXiv:hep-ex/0101041, doi:10.1016/S0370-2693(01)00362-8.
- [108] J. Orloff, A. N. Rozanov, C. Santoni, Limits on the mixing of tau neutrino to heavy neutrinos, *Phys.Lett. B* **550** (2002) 8–15. arXiv:hep-ph/0208075, doi:10.1016/S0370-2693(02)02769-7.
- [109] A. Abada, A. Teixeira, A. Vicente, C. Weiland, Sterile neutrinos in leptonic and semileptonic decays, *JHEP* **1402** (2014) 091. arXiv:1311.2830, doi:10.1007/JHEP02(2014)091.
- [110] A. Abada, D. Das, A. Teixeira, A. Vicente, C. Weiland, Tree-level lepton universality violation in the presence of sterile neutrinos: impact for R_K and R_{π} , *JHEP* **1302** (2013) 048. arXiv:1211.3052, doi:10.1007/JHEP02(2013)048.
- [111] M. Gronau, C. N. Leung, J. L. Rosner, Extending Limits on Neutral Heavy Leptons, *Phys.Rev. D* **29** (1984) 2539. doi:10.1103/PhysRevD.29.2539.
- [112] E. Nardi, E. Roulet, D. Tommasini, Limits on neutrino mixing with new heavy particles, *Phys.Lett. B* **327** (1994) 319–326. arXiv:hep-ph/9402224, doi:10.1016/0370-2693(94)90736-6.

- [113] P. Langacker, D. London, Mixing Between Ordinary and Exotic Fermions, *Phys.Rev.* D38 (1988) 886. doi:10.1103/PhysRevD.38.886.
- [114] S. Antusch, O. Fischer, Non-unitarity of the leptonic mixing matrix: Present bounds and future sensitivities, *JHEP* 1410 (2014) 94. arXiv:1407.6607, doi:10.1007/JHEP10(2014)094.
- [115] A. Atre, T. Han, S. Pascoli, B. Zhang, The Search for Heavy Majorana Neutrinos, *JHEP* 0905 (2009) 030. arXiv:0901.3589.
- [116] G. Czapek, et al., Branching ratio for the rare pion decay into positron and neutrino, *Phys. Rev. Lett.* 70 (1993) 17–20. doi:10.1103/PhysRevLett.70.17.
- [117] B. Aubert, et al., Measurements of Charged Current Lepton Universality and $—V(us)—$ using Tau Lepton Decays to $e^- \nu(e^-)$, $\mu^- \nu(\mu^-)$, $\pi^- \nu(\pi^-)$ and $K^- \nu(K^-)$, *Phys.Rev.Lett.* 105 (2010) 051602. arXiv:0912.0242, doi:10.1103/PhysRevLett.105.051602.
- [118] S. M. Boucenna, S. Morisi, J. W. Valle, The low-scale approach to neutrino masses, *Adv.High Energy Phys.* 2014 (2014) 831598. arXiv:1404.3751, doi:10.1155/2014/831598.
- [119] P. B. Dev, R. Mohapatra, TeV Scale Inverse Seesaw in SO(10) and Leptonic Non-Unitarity Effects, *Phys.Rev.* D81 (2010) 013001. arXiv:0910.3924, doi:10.1103/PhysRevD.81.013001.
- [120] C.-H. Lee, P. Bhupal Dev, R. Mohapatra, Natural TeV-scale left-right seesaw mechanism for neutrinos and experimental tests, *Phys.Rev.* D88 (9) (2013) 093010. arXiv:1309.0774, doi:10.1103/PhysRevD.88.093010.
- [121] M. Drewes, B. Garbrecht, Experimental and cosmological constraints on heavy neutrinos arXiv:1502.00477.
- [122] C. Degrande, O. Mattelaer, R. Ruiz, J. Turner, Fully-Automated Precision Predictions for Heavy Neutrino Production Mechanisms at Hadron Colliders arXiv:1602.06957.
- [123] J. Aguilar-Saavedra, F. Deppisch, O. Kittel, J. W. F. Valle, Flavour in heavy neutrino searches at the LHC, *Phys.Rev.* D85 (2012) 091301. arXiv:1203.5998.
- [124] S. M. Boucenna, S. Morisi, J. W. F. Valle, Radiative neutrino mass in 3-3-1 scheme, *Phys. Rev.* D90 (1) (2014) 013005. arXiv:1405.2332, doi:10.1103/PhysRevD.90.013005.
- [125] S. M. Boucenna, R. M. Fonseca, F. Gonzalez-Canales, J. W. F. Valle, Small neutrino masses and gauge coupling unification, *Phys. Rev.* D91 (3) (2015) 031702. arXiv:1411.0566, doi:10.1103/PhysRevD.91.031702.
- [126] S. M. Boucenna, J. W. F. Valle, A. Vicente, Are the B decay anomalies related to neutrino oscillations?, *Phys. Lett.* B750 (2015) 367–371. arXiv:1503.07099, doi:10.1016/j.physletb.2015.09.040.
- [127] V. V. Vien, A. E. C. Hernandez, H. N. Long, The $\Delta(27)$ flavor 3-3-1 model with neutral leptons arXiv:1601.03300.
- [128] J. Schechter, J. Valle, Neutrinoless Double beta Decay in SU(2) \times U(1) Theories, *Phys.Rev.* D25 (1982) 2951. doi:10.1103/PhysRevD.25.2951.
- [129] F. F. Deppisch, M. Hirsch, H. Pas, Neutrinoless Double Beta Decay and Physics Beyond the Standard Model, *J.Phys.* G39 (2012) 124007. arXiv:1208.0727, doi:10.1088/0954-3899/39/12/124007.
- [130] W. Rodejohann, Neutrino-less Double Beta Decay and Particle Physics, *Int.J.Mod.Phys.* E20 (2011) 1833–1930. arXiv:1106.1334.
- [131] M. Mitra, G. Senjanovic, F. Vissani, Neutrinoless Double Beta Decay and Heavy Sterile Neutrinos, *Nucl.Phys.* B856 26–73.
- [132] F. Simkovic, J. Vergados, A. Faessler, Few active mechanisms of the neutrinoless double beta-decay and effective mass of Majorana neutrinos, *Phys.Rev.* D82 (2010) 113015. arXiv:1006.0571, doi:10.1103/PhysRevD.82.113015.
- [133] J. W. F. Valle, Resonant oscillations of massless neutrinos in matter, *Phys. Lett.* B199 (1987) 432.
- [134] G. C. Branco, M. N. Rebelo, J. W. F. Valle, Leptonic CP violation with massless neutrinos, *Phys. Lett.* B225 (1989) 385.
- [135] N. Rius, J. W. F. Valle, Leptonic CP violating asymmetries in Z0 decays, *Phys. Lett.* B246 (1990) 249–255.
- [136] H. Nunokawa, et al., Resonant conversion of massless neutrinos in supernovae, *Phys. Rev.* D54 (1996) 4356–4363. arXiv:hep-ph/9605301.
- [137] D. Grasso, H. Nunokawa, J. W. F. Valle, Pulsar velocities without neutrino mass, *Phys. Rev. Lett.* 81 (1998) 2412–2415.
- [138] H. Nunokawa, S. J. Parke, J. W. Valle, CP Violation and Neutrino Oscillations, *Prog.Part.Nucl.Phys.* 60 (2008) 338–402. arXiv:0710.0554, doi:10.1016/j.pnpnp.2007.10.001.
- [139] P. Adamson, et al., First measurement of electron neutrino appearance in NOvA arXiv:1601.05022.
- [140] D. Aristizabal Sierra, M. Tortola, J. Valle, A. Vicente, Leptogenesis with a dynamical seesaw scale (2014). arXiv:1405.4706, doi:10.1088/1475-7516/2014/07/052.
- [141] S. M. Boucenna, S. Morisi, Q. Shafi, J. W. F. Valle, Inflation and majoron dark matter in the seesaw mechanism, *Phys.Rev.* D90 (2014) 055023. arXiv:1404.3198, doi:10.1103/PhysRevD.90.055023.
- [142] G. F. Smoot, See Saw Inflation / Dark Matter / Dark Energy / Baryogenesis arXiv:1405.2776.
- [143] M. Guzzo, et al., Status of a hybrid three-neutrino interpretation of neutrino data, *Nucl. Phys.* B629 (2002) 479–490. arXiv:hep-ph/0112310 v3 KamLAND-updated version.
- [144] T. Ohlsson, Status of non-standard neutrino interactions, *Rept. Prog. Phys.* 76 (2013) 044201. arXiv:1209.2710, doi:10.1088/0034-4885/76/4/044201.
- [145] O. G. Miranda, H. Nunokawa, Non standard neutrino interactions: current status and future prospects, *New J. Phys.* 17 (9) (2015) 095002.
- [146] A. de Gouvea, K. J. Kelly, Non-standard Neutrino Interactions at DUNE this volume, 2016.
- [147] O. Miranda, M. Tortola, J. Valle, Are solar neutrino oscillations robust?, *JHEP* 0610 (2006) 008. arXiv:hep-ph/0406280, doi:10.1088/1126-6708/2006/10/008.
- [148] F. J. Escrivuela, O. G. Miranda, M. A. Tortola, J. W. F. Valle, Constraining nonstandard neutrino-quark interactions with so-

lar, reactor and accelerator data, Phys. Rev. D80 (2009) 105009, [Erratum: Phys. Rev.D80,129908(2009)]. arXiv:0907.2630, doi:10.1103/PhysRevD.80.129908, 10.1103/PhysRevD.80.105009.